

# Development of Assimilation Methods for Near-Shore Spectral Wave Models

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## LONG-TERM GOAL

To develop methods for assimilating in-situ observations into near-shore spectral wave models so as to improve predictions, and enable model-based interpretation of in-situ data.

## OBJECTIVES

The objectives of this program are: (1) development of an assimilation capability for in-situ observations based on the SWAN model Booij, Ris & Holthuijsen (1999); and (2) validation of the approach using available data.

## APPROACH

The ability to assimilate observations of waves into near-shore spectral wave models will allow accurate prediction of the wave conditions for an entire coastal region. Synthetic-aperture radar (SAR) and other remote sensing techniques offer large area coverage and high spatial resolution and so directional spectrum information can be extracted. For these reasons, methods for assimilating SAR-image data were developed under the ONR Advanced Wave Prediction Program (AWPP). Data from in-situ sensors have their own strengths and limitations. By their nature they lack wide area coverage and provide data for only a single spatial location or in sparse arrays. The data are often incomplete; they usually only can provide frequency spectrum information, and sometimes only integral measures of the directional spectrum. The proposed program will develop the necessary methods to assimilate in-situ observations of this sort.

The assimilation procedure is based on the SWAN model (Booij *et al.* 1999), a near-shore wave-action-balance model which can predict the evolution of the wave spectrum in coastal regions. The wave-action spectral balance is expressed as

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}(C_x N) + \frac{\partial}{\partial y}(C_y N) + \frac{\partial}{\partial \sigma}(C_\sigma N) + \frac{\partial}{\partial \theta}(C_\theta N) = \frac{S}{\sigma} \quad (1)$$

where  $x$  and  $y$  are spatial position variables, and  $\sigma$  and  $\theta$  are wave frequency and direction variables for the action spectrum. The vectors  $\mathbf{x} = (x, y)$  and  $\mathbf{s} = (\sigma, \theta)$  will be used to represent spatial and spectral position, respectively.  $N(\mathbf{x}, \mathbf{s}, t)$  is the action spectral density defined as

$$N(\mathbf{x}, \mathbf{s}, t) = \frac{E(\mathbf{x}, \mathbf{s}, t)}{\sigma(\mathbf{x}, \mathbf{s})} \quad (2)$$

where  $E$  is the energy spectral density. The source term on the right-hand side of (1) is described in

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detail in Booij *et al.* (1999) and includes the effects of wind growth and energy transfer in the spectrum due nonlinear wave-wave interactions (resonant triad and quartet interactions). Significant additional contributors to the source term are various processes by which wave energy is dissipated. These include white-capping, bottom friction and depth-induced breaking.

Assimilation is the process of determining the set of model inputs required so as to produce predictions which are in agreement with observations (Le Dimet & Talagrand 1986 and Bennett 1992). In the context of the SWAN model, this can mean determination of the incident wave spectrum for a near-shore region based on wave-spectrum observations in the interior of the region. In this case, the formal adjoint to the SWAN-model equation is solved for the adjoint wave-action spectrum. The adjoint action balance equation is

$$\frac{\partial A}{\partial t} + C_x \frac{\partial A}{\partial x} + C_y \frac{\partial A}{\partial y} + C_\sigma \frac{\partial A}{\partial \sigma} + C_\theta \frac{\partial A}{\partial \theta} = \frac{1}{\sigma} \frac{\delta \mathcal{S}}{\delta N} A + \frac{1}{\sigma} (E - \hat{E}_i) \sum_i \delta(t - t_i) \quad (11)$$

where  $A(\mathbf{x}, \mathbf{s}, t)$  is the adjoint to the wave-action spectral density, and the right-hand side contains the first variation of the source function with respect to the action spectral density  $N$ , and a term containing observations of the wave spectrum at discrete times. From the adjoint solution on the incident-wave boundary, a correction to the incident wave spectrum can be calculated which will result in closer agreement between the predictions and observations. This correction is applied iteratively until the difference between the predicted and observed wave spectrum is minimized.

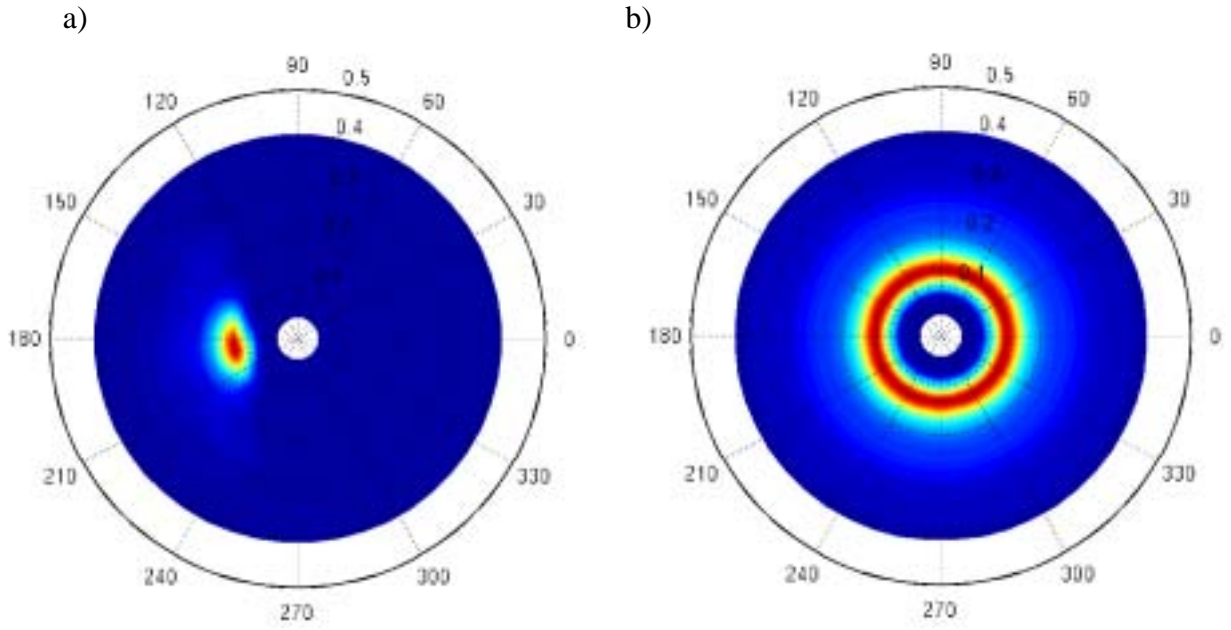
In Walker (2001), the ability to reconstruct the incident wave spectrum from a single directional spectrum observation in the interior of the region is demonstrated. The results showed that the integral measures of the estimated wave spectrum (significant wave height, dominant wave direction and frequency) for the observation location and another near-shore location were accurate to within a few percent. The estimated incident wave spectrum was less accurate (~10% error in integral measures) due primarily to the neglect of the nonlinear wave-wave interactions in the adjoint equations. In that study, the wave spectrum observations were obtained from a forward prediction SWAN model; this was done to ensure that it was a true test of variational assimilation procedure.

## WORK COMPLETED

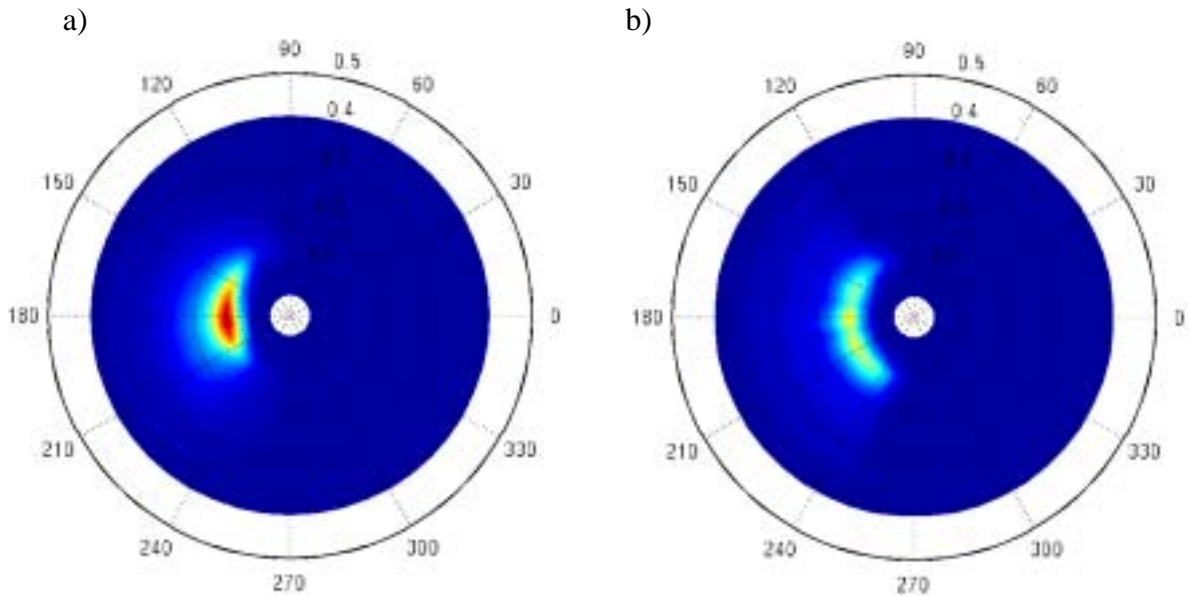
During FY 01, a variational approach for assimilating non-directional frequency spectrum data observations into the SWAN model was developed.

## RESULTS

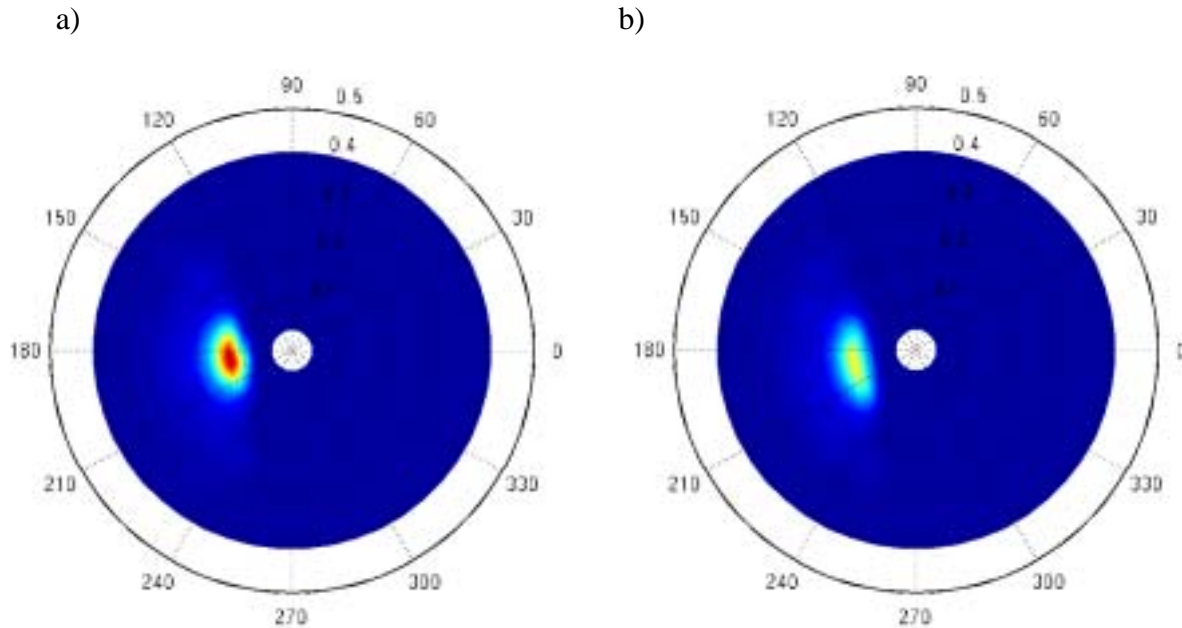
Work on this program started late in the fiscal year, and the initial task was to develop an approach using the existing adjoint and forward SWAN models for assimilation of a single-spatial-point wave frequency spectrum. The case examined was for the region around The USACE Field Research Facility in Duck, NC. For this study, the SWAN model was run using a prescribed input directional spectrum at the open ocean boundary, assuming steady state behavior. A ‘synthetic’ observation for a single, near-shore location (that of the FRF 8m pressure array) was extracted. This ‘observation’, a directional spectrum, was integrated to produce a non-directional frequency spectrum which was used for assimilation. Figure 1a shows the directional spectrum (the synthetic observation) obtained from the SWAN model run. The lack of directional information in the spectrum to be assimilated is accommodated by assimilating an isotropic directional spectrum which has the same frequency spectrum.



**Figure 1. Wave spectrum at FRF 8m array location: a) frequency–direction spectrum ( $H_{mo} = 1.56$  m,  $f_D = 0.165$  Hz,  $\theta_D = 187^\circ$ ); b) non-directional (isotropic) frequency spectrum calculated from that in a) and used in assimilation ( $H_{mo} = 1.56$  m,  $f_D = 0.165$  Hz).**



**Figure 2. Incident wave spectrum at ocean boundary for SWAN model: a) actual frequency–direction spectrum ( $H_{mo} = 2.03$  m,  $f_D = 0.169$  Hz,  $\theta_D = 181^\circ$ ); b) estimated frequency–direction spectrum based on assimilation of non-directional (isotropic) frequency spectrum ( $H_{mo} = 1.86$  m,  $f_D = 0.169$  Hz,  $\theta_D = 188^\circ$ ).**



**Figure 3. Wave spectrum at FRF 8m array location: a) actual frequency–direction spectrum ( $H_{mo} = 1.56$  m,  $f_D = 0.165$  Hz,  $\theta_D = 187^\circ$ ); b) estimated frequency– direction spectrum based on assimilation of non-directional (isotropic) frequency spectrum the ( $H_{mo} = 1.42$  m,  $f_D = 0.168$  Hz,  $\theta_D = 191^\circ$ ).**

Figure 3 shows the results for the location of the assimilated spectrum observation. Again the agreement is good between the estimated directional spectrum (Figure 3b) and the ‘true’ directional spectrum (Figure 3a), with a 9% error in significant wave height and substantially smaller errors in dominant wave frequency and direction. Again the directional spread in the estimated spectrum is the main discrepancy, which is again unsurprising given the non-directional nature of the input data (Figure 1b).

## IMPACT/APPLICATION

Achieving the overall objectives of the program will result in an improved prediction capability for near-shore waves, allowing readily available data to be used effectively.

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